Ray-Plane Intersection

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1 Introduction

In this document we'll calculate the location of the intersection point between a ray and a plane.

2 The setting

Given, we have

- Ray, starting at point R, with direction, given by the vector $\vec{r}$
- Plane, defined by an adjacent point A and a normal vector $\vec{n}$

For clarity and simplicity in calculations, we'll have $\vec{r}$ and $\vec{n}$ normalized, that is $|\vec{r}| = 1$ and $|\vec{n}| = 1$.

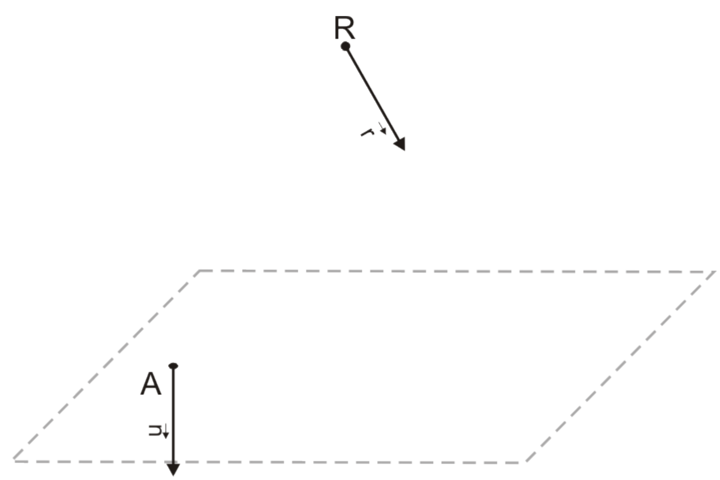

Figure 1: Setting

3 Solution

We'll start our search by supposing there is an intersection point, S.
This point lies somewhere along the ray. We'll describe it by defining a parameter d, which is the distance between R and S.
In other words

$$S = R + \vec{r}d$$

Now we have a simple scalar value, which will give us the intersection point.
We'll look for some simple figure, that is fully defined and contains d.
Let's drop a perpendicular from R down to a point on the plane and call that point P. The orientation of this perpendicular is $\vec{n}$ and its length is the distance between point R and the plane.

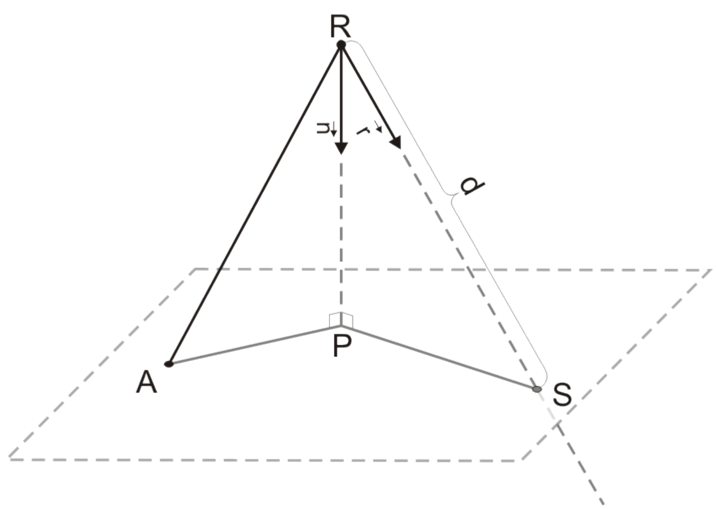

Figure 2: Concept

From $\triangle APR(\angle APR = \pi)$

$$RP = RA.\cos(\angle ARP)$$

but $\vec{n} \cdot \vec{RA} = RA.\cos(\angle ARP)$, therefore

$$RP = \vec{n} \cdot \vec{RA}$$

From $\triangle SPR(\angle SPR = \pi)$, $RP = d.\cos(\angle PRS)$, or

$$d = \frac{RP}{\cos(\angle PRS)}$$

but $\vec{n} \cdot \vec{r} = \cos(\angle PRS)$, therefore

$$d = \frac{\vec{n} \cdot \vec{RA}}{\vec{n} \cdot \vec{r}}$$

Finally, we have

$$S = R + \vec{r}\left(\frac{\vec{n} \cdot \vec{RA}}{\vec{n} \cdot \vec{r}}\right), \vec{n} \cdot \vec{r} \neq 0$$

4 Notes

4.1 Restrictions

In the result we have division by $\vec{n} \cdot \vec{r}$. Of course, to be mathematically correct, we should at least ensure it's not zero. But what Math is actually telling us by that, is if the ray forms right angle with the plane normal (meaning the ray and the plane are parallel), then $d = \infty$, that is, the intersection point would be at infinite distance because the ray wouldn't intersect with the plane.

4.2 Distance between a point and a plane and orientation of $\vec{n}$

Generally, $|RP|$ gives us the distance between the ray initial point and the plane, and the sign of RP tells us in which of the two half spaces, split by the plane, is the point R - it's positive if R and the normal orientation are in different half spaces (as in our setting) and negative otherwise.

4.3 Planes "behind" the ray

We can pick planes that intersect on the negative side of the ray, that is, on the same line as the ray, but "behind" point R. In this case, the distance, d will simply be negative.