Ray-Triangle Intersection

http://morroworks.com

1 Introduction

In this document we'll calculate the intersection point between a ray and a triangle by

- Finding the coordinates of the point of intersection between the ray and the plane on which the given triangle lies
- Determining whether this point lies within the triangle by calculating its barycentric coordinates.

For solving each of these steps, we'll refer to the documents "Ray-Plane Intersection" and "Barycentric Coordinates".

2 Setting

Given, we have

- Ray, starting at point R, with direction, given by $\vec{r}$, where $|\vec{r}| = 1$.
- Triangle, defined by the points A, B, C.

3 Solution

A triangle is a part of a larger, in fact infinite geometry structure - the plane it lies on. Unless the ray is parallel to that plane, it would intersect it at some point, and if that point is within the boundaries of the triangle, we have a hit.

We'll first calculate the intersection point between the ray and the plane and call that point S. In order to do that, we'll need a vector that is normal to the plane, which we'll name $\vec{n}$.
Secondly, we'll calculate this point's barycentric coordinates in aspect to the coordinate system defined by the triangle. Let's call these coordinates u and v.

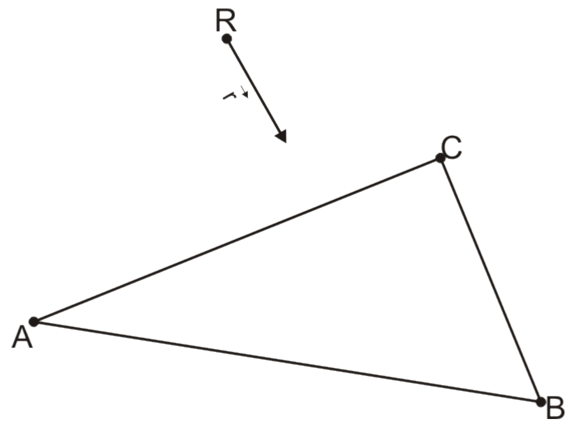

Figure 1: Setting

3.1 Calculating intersection point location

Finding the intersection point between a ray and a plane is thoroughly covered in the document "Ray-Plane Intersection".

For this task we'll need the ray's initial point and its normalized orientation vector, which we have by definition. For the plane, we'll need a point, lying on it - let's take point A, and a vector which is normal to the plane.
Such a vector can easily be calculated by cross multiplying two vectors that lie on the plane, in this case any two of the triangle's arms. Let's take $\vec{AC}$ and $\vec{AB}$.

$$\vec{n} = \vec{AC} \times \vec{AB}$$

and let's normalize $\vec{n}$, so that

$$|\vec{n}| = 1$$

Now, with these in hand we can calculate the intersection point between the ray and the triangle's plane. That is

$$S = R + \vec{r}\left(\frac{\vec{n} \cdot \vec{RA}}{\vec{n} \cdot \vec{r}}\right), \vec{n} \cdot \vec{r} \neq 0$$

Note that if $\vec{n} \cdot \vec{r} = \vec{0}$ that would mean the ray is perpendicular to the plane normal, in other words, the ray and the plane are parallel, in which case the intersection point goes to infinity along the ray.

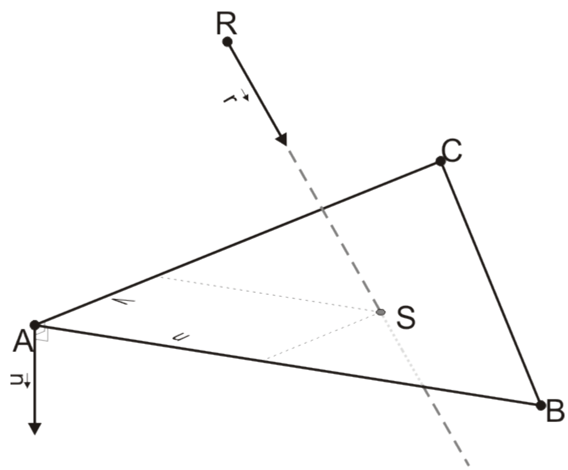

Figure 2: Concept

As we've got the intersection point calculated, the only remaining thing is to check whether it's within the triangle.

3.2 Calculating intersection point's barycentric coordinates and determining whether it lies within the triangle

Finding a point's barycentric coordinates is thoroughly covered in the document "Barycentric Coordinates".

Having a point of intersection between the ray and the triangle's plane, let's determine whether this point is inside the boundaries of the triangle.
We'll do that by computing the point's barycentric coordinates (u, v) in relation to the coordinate system with barycenter A and axes $\vec{AB}$ and $\vec{AC}$.

$$u = \frac{\left(\vec{AB} \cdot \vec{AS}\right)\left(\vec{AC} \cdot \vec{AC}\right) - \left(\vec{AB} \cdot \vec{AC}\right)\left(\vec{AC} \cdot \vec{AS}\right)}{\left(\vec{AB} \cdot \vec{AB}\right)\left(\vec{AC} \cdot \vec{AC}\right) - \left(\vec{AB} \cdot \vec{AC}\right)\left(\vec{AB} \cdot \vec{AC}\right)}$$

$$v = \frac{\left(\vec{AB} \cdot \vec{AS}\right)\left(\vec{AC} \cdot \vec{AC}\right) - \left(\vec{AB} \cdot \vec{AC}\right)\left(\vec{AC} \cdot \vec{AS}\right)}{\left(\vec{AB} \cdot \vec{AB}\right)\left(\vec{AC} \cdot \vec{AC}\right) - \left(\vec{AB} \cdot \vec{AC}\right)\left(\vec{AB} \cdot \vec{AC}\right)}$$

With the restrictions
$\vec{AB} \neq \vec{0}$ - in order to have a valid $\vec{AB}$
$\vec{AC} \neq \vec{0}$ - in order to have a valid $\vec{AC}$

$\vec{AB} \neq \vec{AC}$ - in order to have a valid triangle

Finally, with (u, v) calculated, we can determine if the point lies inside of the triangle, if the following conditions are met:

$$u > 0$$
$$v > 0$$
$$u + v < 1$$

In addition to these, if $u = 1$, then the point lies on AC, if $v = 1$ it lies on AB and if $u + v = 1$, it lies on BC.

4 Notes

4.1 Triangles "behind" the ray

We can pick triangles that intersect on the negative side of the ray, that is, on the same line as the ray, but "behind" point R. In this case, the distance, d will simply be negative.

4.2 Useful data and triangle filtering

Note that in the process of calculating S, as explained in "Ray-Plane Intersection" we've also calculated the distance between R and S, the distance between R and the plane, plus the orientation of the plane normal.
For example, we can choose to only pick triangles ahead of the ray, or filter triangles, which are oriented backwards.

4.3 Triangle orientation

Because The plane normal, however was calculated via $\vec{AC} \times \vec{AB}$

If plane orientation is to be taken into consideration, then the order in which to carry the multiplication of $\vec{AC}$ and $\vec{AB}$ for the calculation of $\vec{n}$ is of importance, because reversing the order will also reverse the orientation of the resulting normal vector.
In practice, multiplication is usually performed in only one of the two cases with consideration of the order of the triangle's vertices - clockwise or counter clockwise. In our setting, we use counter clockwise order, multiplying $\vec{AC}$ as first argument, by $\vec{AB}$ as second.

4.4 Barycentric coordinates and further usage

The barycentric coordinates we've got are of much more value than just for determining if the point is inside the triangle. They say exactly where the point is in terms of the triangle's space, in other words exactly where in the triangle it is, just as ordinary (x, y) coordinates in Cartesian coordinate system would describe a point.

For example, these coordinates can be used to easily interpolate (as well, of course, as extrapolate) a value for the intersection point, based on values we have for the triangle's vertices. This is frequently used in graphics, for calculating, say, color or texture coordinate value for a point in a triangle.
For example, suppose V_a, V_b and V_c are values at A, B and C respectively. We can calculate the value at S as

$$V_s = V_a + uV_b + vV_c$$

See "Barycentric coordinates" for further information.